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SEA LEVEL RISE AND THE GEOID: FACTOR ANALYSIS APPROACH

CRECIMIENTO DEL NIVEL DEL MAR Y EL GEOIDE: ENFOQUE DEL ANÁLISIS FACTORIAL

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Abstract

Sea levels are rising around the world, and this is a particular concern along most of the coasts of the United States. A 1989 EPA report shows that sea levels rose 5-6 inches more than the global average along the Mid-Atlantic and Gulf Coasts in the last century. The main reason for this is coastal land subsidence. This sea level rise is considered more as relative sea level rise than global sea level rise. Thus, instead of studying sea level rise globally, this paper describes a statistical approach by using factor analysis of regional sea level rates of change. Unlike physical models and semi-empirical models that attempt to approach how much and how fast sea levels are changing, this methodology allows for a discussion of the factor(s) that statistically affects sea level rates of change, and seeks patterns to explain spatial correlations.

Keywords: Sea level, the geoid, factor analysis.

Resumen

Los niveles del mar crecen alrededor del mundo, y este es un problema que afecta a la mayoría de las costas en los Estados Unidos. Un reporte de 1989 de la Agencia de Protección Ambiental de los Estados Unidos (EPA, por sus siglas en inglés) muestra que el nivel del mar creció de 5 a 6 pulgadas más que el promedio global en el Atlántico Medio y la costa del Golfo en el siglo pasado. La principal razón para esto es el hundimiento de la tierra costera. Este crecimiento del nivel del mar es considerado más como uno relativo antes que global. Luego, en lugar de estudiar el crecimiento global del nivel del mar, este artículo describe un enfoque estadístico usando análisis factorial del cambio en las razones del nivel del mar regional. Contrario a los modelos físicos o semiempíricos, que tratan de aproximar qué tanto y qué tan rápido cambian los niveles del mar, esta metodología permite una discusión de los factores que afectan estadísticamente las razones de cambio del nivel del mar, y busca patrones que expliquen correlaciones espaciales.

Palabras clave: Nivel del mar, el geoide, análisis factorial.

Mathematics Subject Classification: 86A32, 62P.

1 Introduction

Most scientists consider climate change to be a serious environmental threat (IPCC 2007). Climate change, whether from natural or anthropogenic causes, is evidenced by increased rates of sea level rise, increased atmospheric and ocean temperatures, changes to precipitation amounts and patterns, a possible subtropical desert expansion (Lu et al. 2007), thermal expansion of ocean water, and glaciers melting. Other effects of climate change are evidenced by severe weather events including heat waves, droughts (Dai 2010), and heavy rainfall. Some potential effects of climate change include species extinctions due to shifting temperature regimes, the threat to food security because of extreme weather patterns (Battisti & Naylor 2009), and habitat losses because of coastal inundation due to higher rates of sea level rise.

Sea level rise demands more attention in coastal areas. One reason is that about 10% of the world's population lives in low-lying coastal areas with elevations less than 10 meters above current mean sea level (FitzGerald et al., 2008). The current global rate of sea level rise is nearly 3.0 mm/year (Rahmstorf, 2007a; Nicholls & Cazenave, 2010). Satellite observations show the rate of sea level rise varies across the globe (http://sealevel.colorado.edu/content/regional-sea-level-time-series). Thus, the study of sea level rise is an important component of earth science research.

Cartographers and geodesists, those who study the measurement of the size and shape of the Earth, are interested in sea level as an elevation datum. This datum is called the geoid, which is defined as the equipotential gravity surface of the Earth, and theoretically best fits global mean sea level in ocean areas (Hofmann-Wellenhof & Moritz, 2006). Hence, the rate of change in mean sea level directly affects changes to the geoid and the elevation datum used as the reference for topographic mapping.

Many methods have been used in sea level rise modeling. These methods can be divided into two categories: physical models, based on the conservation of mass (global water mass and ice mass measurements), and semi-empirical models, studying measured rates of change of sea level and measured changes in global temperatures along with the error estimates of measurements to predict future trends (Rahmstorf 2012). These two approaches are complementary. For example, no one really understands the dynamics of each and every glacier, so it is quite difficult to calculate melting glaciers from physical models, hence the use of semi-empirical methods described in the majority of studies of sea level rise. This paper introduces a different approach by using factor analysis of regional sea level rates of change as a statistical analysis tool. Instead of answering the question of how much and how fast sea levels are changing, this paper computes and discusses which mathematical factor statistically affects sea level rates of change and seeks patterns to explain spatial correlation. The paper also seeks to hypothesize that any insights into the factors influencing sea level change also apply to the changes to the geoid.

2 Background

There are two types of sea level rise. One is called global sea level rise. The cause of global sea level rise is basically rising temperatures. According to Roemmich (1992), thermal expansion of seawater and melted glaciers are increasing results from warming. Additionally, a 2009 EPA report said that potential changes in polar ice sheet flow may be another factor which causes global sea level rise (Williams et al. 2009). The Intergovernmental Panel on Climate Change (IPCC) reported that sea levels have risen approximately 4.8 $\sim$ 8.8 inches (12 $\sim$ 22 cm) around the world during last century (IPCC, 2007). Global sea levels rose at an average rate of 1.8 mm per year between 1961 and 2003 (1.3 $\sim$ 2.3 mm), and there was a much faster rate between 1993 and 2003 (Williams et al. 2009). The IPCC (2007) estimated that the global mean sea level will rise by 7.2 $\sim$ 23.6 inches (18 $\sim$ 59 cm) by 2100 (Figure 1). The CU Sea Level Research Group (SLRG) at the University of Colorado defines the global mean sea level as "the area-weighted mean of all of the sea surface height anomalies measured by the altimeter in a single, 10-day satellite track repeat cycle". The SLRG at CU also argues that the global mean sea level can be thought of as the eustatic sea level, which represents the level if all the water in the ocean is based on a single basin.

Another type is local sea level rise, often known as relative sea level rise, which is very important when studying coastal areas. It refers to the change in sea levels relative to the elevation of the land, and relative sea level rise includes the effect of both global sea level rise and vertical movements of the land (Williams et al. 2009). For instance, relative sea level rise along the coastal areas of the Gulf of Mexico is caused by the global rise in ocean levels and land subsidence resulting from natural and human-induced changes (Montagna et al. 2007). From natural view, the coastal plain was built by deposited, estuaries, coastal, and sediments. The thickness of mud and sand is compressing under their own weight at a rate of about 0.05 mm/year (Paine 1993). In addition, land subsidence is a human-induced result from extraction of subsurface fluids (i.e. oil, gas, water). Montagna et al. (2007) suggested that the highest rates of land subsidence are correlated with oil, gas, and groundwater production

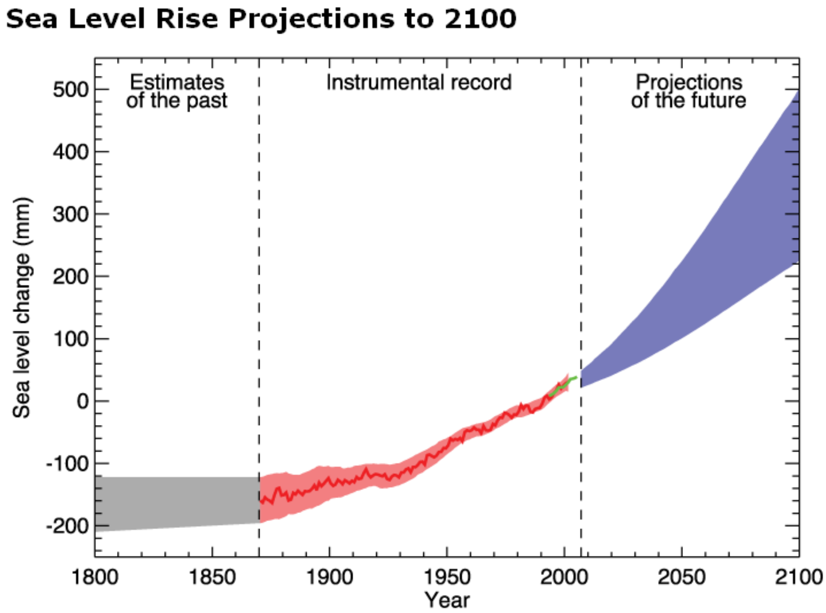


Figure 1: Past and projected global average sea level. The gray shaded area shows the estimates of sea level change from 1800 to 1870 when measurements are not available. The red line is a reconstruction of sea level change measured by tide gauges with the surrounding shaded area depicting the uncertainty. The green line shows sea level change as measured by satellite. The purple shaded area represents the range of model projections for a medium growth emissions scenario (IPCC SRES A1B). Source: IPCC (2007).

in South Texas coasts; however, Dokka (2006) proposed that a significant cause of subsidence is faulting in the Gulf of Mexico.

Figure 2 (Montagna et al. 2007) visually shows how a rise of 2, 4, and 6 meters respectively in sea level would result in inundations of the Corpus Christi Bay area. With a rise of less than 4 meters, barrier islands that exist today would be completely gone. With only a 2-meter rise from current sea level, the lower Nueces Delta would be submerged, and the entire delta would be submerged with a 6 meters rise of sea level. This figure does not give us a realistic view or shape for future shorelines because it did not include variable factors (i.e. waves, currents, and human activities) in this case. But just by this view, there would be massive losses of marsh habitats in the bays. This is why studying and understanding what kind of factors lead to sea level rise is imperative.

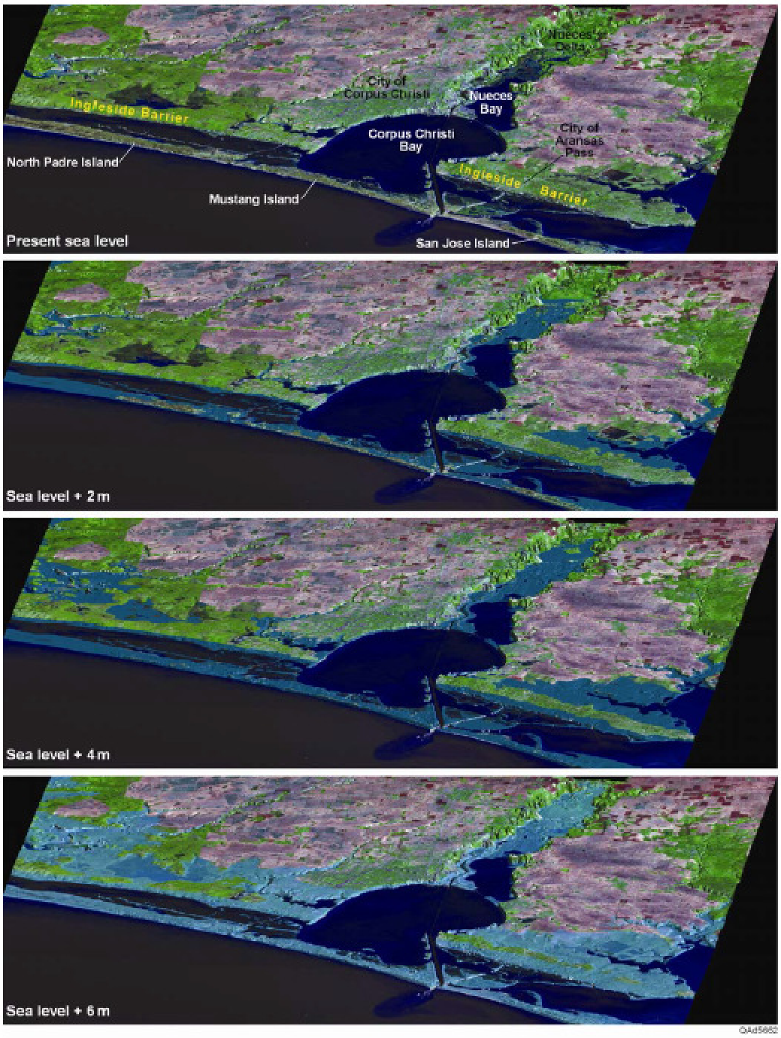


Figure 2: Perspective view of inundation of the Corpus Christi Bay area by sea level rise (Montagna et al. 2007).

3 Data

Datasets were downloaded from the University of Colorado Sea level research Group[1] (Figure 3). The sea level data was recorded from 1992 to 2011 (Figure 4a). These datasets did not correct GIA, but applied the

[1]Website http://sealevel.colorado.edu/content/regional-sea-level-time-series.

inverted barometer (IB) approach, which is the traditional static model, and also included seasonal signals.

These downloaded datasets were fixed by using a running average (also known as a moving average) in this study. The cumulative running average (CRA) is typically the unweighted average of the sequence of i mean sea level values $x_1, x_2, \ldots, x_i$ upto 2011:

$$CRA_i = \frac{(x_1 + x_2 + \ldots + x_i)}{i}.$$

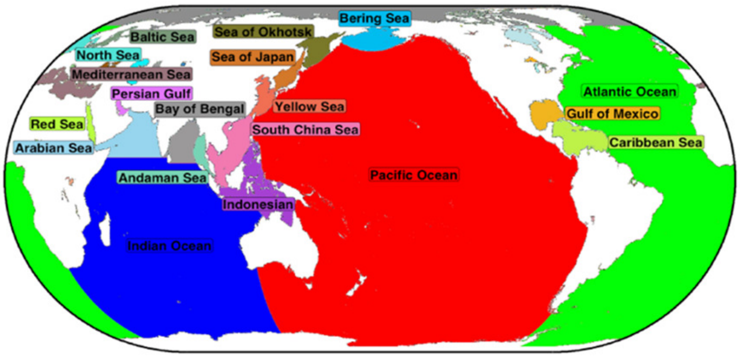
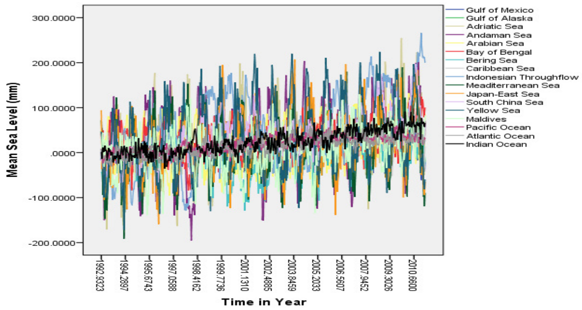


Figure 3: Data from each region represented in the world map, http://ibis.grdl.noaa.gov/SAT/SeaLevelRise/LSA_SLR_timeseries_regional.php.

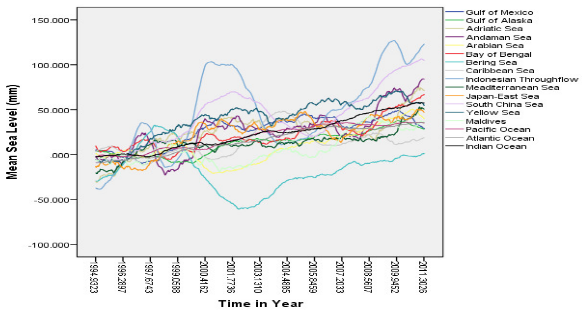
In principle, a prediction lies on the original regression line, and an increase in the strength of correlation (either positive or negative) is expected. If it is a good prediction, the correlation coefficient after the extra order pair added to the data should be stronger than the original coefficient. Unfortunately, this is not likely to happen due to the mutative trend of coefficient. A running average method uses the i pairs of data to calculate the regression equation and correlation coefficient, and increases i by 1 each step, and repeats the process until reaching the suitable number of predictions. Therefore, there is a higher estimate from the mutative trend, and the prediction will be much smoother (Figure 4b and 4c).

4 Method

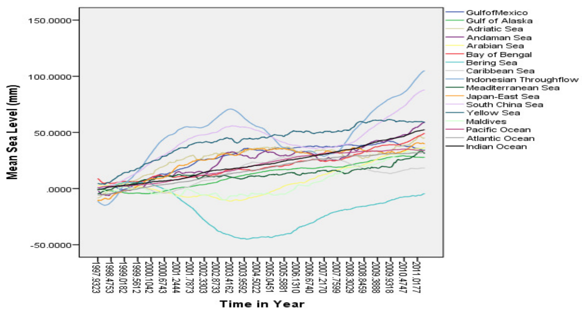
Using a running average of mean sea level data, the factor analysis approach was applied in this study. Factor analysis gives a better understanding among variables during the process. Firstly, the correlation



(a) Sea level dataset 1992–2011.



(b) 2 years running averages.



(c) 5 years running averages.

Figure 4: Level variations.

matrix was obtained by using IBM SPSS Statistics 20. The extraction method used principal component analysis, and the rotation method used varimax with Kaiser Normalization. Then the four criteria-eigenvalue, variance, scree plot and residuals-were tested. Table 1 represents the communalities, and it indicates that all variables are $> .9$. Thus, this analysis is fairly reliable. Finally, the number of factors to be retained was determined. Since components with eigenvalues greater than 1 should be retained, the first three components satisfied this criterion.

	Initial	Extraction
Gulf of Mexico	1.000	.976
Gulf of Alaska	1.000	.981
Adriatic Sea	1.000	.953
Andaman Sea	1.000	.967
Arabian Sea	1.000	.994
Bay of Bengal	1.000	.985
Bering Sea	1.000	.973
Caribbean Sea	1.000	.970
Indonesian Throughflow	1.000	.986
Meaditerranean Sea	1.000	.944
Japan-East Sea	1.000	.981
South China Sea	1.000	.977
Yellow Sea	1.000	.963
Maldives	1.000	.983
Pacific Ocean	1.000	.996
Atlantic Ocean	1.000	.991
Indian Ocean	1.000	.997

Table 1: Communalities of mean sea level.

The total variance of the first three components is 97.748% (Table 2). However, if we take a careful look at the initial analysis, only the first component was strong enough to be retained. In this case, a principal component analysis was conducted to keep three components and utilize the varimax rotation. Inclusion of two components increases the model fit, and three components are fairly strong to be retained. After rotation, the first component accounted for 35.237%, the second for 32.655%, and the third for 29.856%.

Comp.	Initial Eigenvalues			Extraction Sums of Sqd. Loadings			Rotation Sums of Sqd. Loadings		
	Total	% Variance	Cumul.%	Total	% Variance	Cumul.%	Total	% Variance	Cumul.%
1	12.872	75.717	75.717	12.872	75.171	75.717	5.990	35.237	35.237
2	2.560	15.058	90.775	2.560	15.058	90.775	5.551	32.655	67.892
3	1.185	6.973	97.748	1.185	6.973	97.748	5.076	29.856	97.748
4	.162	.954	98.70						
5	.118	.694	99.39						
6	.038	.224	99.62						
7	.022	.132	99.75						
8	.011	.064	99.81						
9	.009	.052	99.98						
10	.007	.043	99.91						
11	.005	.028	99.94						
12	.004	.024	99.96						
13	.003	.018	99.98						
14	.001	.009	99.99						
15	.001	.007	99.99						
16	.000	.002	100.0						
17	.000	.000	100.0						

Table 2: Table of total variance for three components solution.

The scree plot (Figure 5) was then evaluated and shows that after component 3, the eigenvalues level off. The process of analysis for determining the appropriate number of components to retain was quite reliable according to the four criteria.

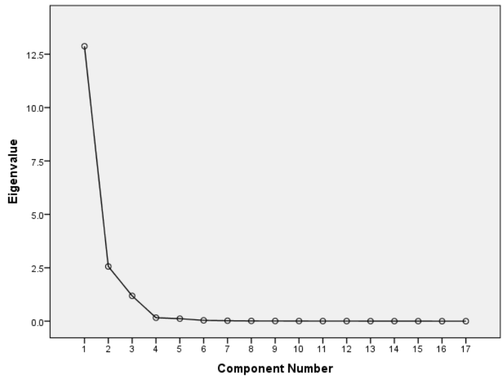


Figure 5: Scree plot.

5 Results

The last step was to interpret each retained component. Table 3 indicates the factor loadings for the rotated components. Only the factor loadings over .7 were concerned in this study. Component 1 consisted of four of seventeen variables: the Arabian Sea, the Bay of Bengal, the Maldives, and the Indian Ocean. These variables had positive loadings. The Arabian Sea and Bay of Bengal are marginal seas of the Indian Ocean. The Maldives Republic is the lowest country in the world. According to EGM08 (Earth Gravity Model of 2008) map, the geoid of these four variables is much lower than the others. Thus, component 1 addressed the geoid. Component 2 included the Gulf of Mexico, the Caribbean Sea, the East Sea (also known as the Sea of Japan), and the Bering Sea. Among of these four variables, the loading of the Bering Sea was negative. The Bering Sea is marginal sea of the Pacific Ocean, and the circulation of this area is also affected by the Arctic Ocean. The currents of the Gulf of Mexico, the Caribbean Sea and the East Sea are warmer current. This second component represented circulation (especially indicate current). Component 3's interpretation is in question. Indonesian throughflow has a long-term history of tectonic changes. These changes were not just zone collision (i.e. Asian-Australia

	Component		
	1	2	3
Gulf of Mexico	.372	.776	.484
Gulf of Alaska	.691	.603	.373
Adriatic Sea	.181	.648	.708
Andaman Sea	.523	.525	.646
Arabian Sea	.988	.020	.127
Bay of Bengal	.802	.354	.465
Bering Sea	.422	-.849	-.272
Caribbean Sea	.265	.947	.068
Indonesian Throughflow	.251	.148	.949
Meaditerranean Sea	.645	.249	.683
Japan-East Sea	.235	.732	.624
South China Sea	.309	.310	.886
Yellow Sea	.470	.609	.609
Maldives	.952	-.003	.276
Pacific Ocean	.662	.667	.335
Atlantic Ocean	.654	.635	.399
Indian Ocean	.758	.458	.460

Table 3: Factor loadings for rotated component matrix.

collision zone) and mountain building, but also included basins extensions and new ocean basins' formation in eastern Indonesia (Kuhnt, W., et al. 2004). The Adriatic Sea is located in the collision zone between the African and the European plates (Favali, P., et al. 1993). So, component 3 may be influenced by the tectonic changes. Additionally, there is an attentionthat the areas with the loadings of the two components which were relatively high are mostly the places where oil spills have occurred, but this hypothesis has not been studied in this research.

6 Discussion

Factor analysis is not designed to clearly represent each factor, but is based on qualities of evidence to identify each factor. Thus, some factors represented here may be different from others' analysis, but the same method is applied. There are always some things we are not able to determine with certainty. It is east to apply the methodology precisely,

	Component	
	1	2
Packer Channel	0.986	0.094
Bob Hall Pier	0.734	0.64
Rockport	0.971	0.099
Galveston Pleasure Pier	0.993	0.049
Galveston Pier 21	0.97	−0.022
Freeport	0.994	0.091
Rincon del San Jose	0.905	0.391
Port Aransas	0.906	0.335
Sabine Pass	0.881	0.428
Port Isabel	0.158	−0.928
S. Padre Island Coast Guard Sta.	0.615	0.659

Extraction Method: Principal component analysis.
Rotation Method: Varimax with Kaiser Normalization.
Rotation converged in 3 iterations.

Table 4: Factor loading for rotated component matrix (Sadovski, A. et al. 2010).

but interpretation varies among interdisciplinary branches of science.

According to a poster presentation by A. Sadovski et al. (Table 4, 5), many factors of sea level rise are unexplained in the analysis of Texas coast, but more factors are identifiable in the pattern of regional areas. In studying regional and local areas, different kinds of datasets (i.e. wind data, salinity data, temperature data, etc.) should be added. This will provide a more precise indication of which factors have a great impact in local areas.

The study of the mean sea level should not just focus on global, but regional or local areas, as well. Studying global sea level changes is helpful for finding and learning about changing patterns, but regional and local studies will reveal more specific factors that cause sea level rise. Knowing the patterns and factors which affect sea levels will result in more accurate predictions of changes along the coasts, and, ultimately, better means by which to plan for, or avoid, catastrophes due to inundation.

	Initial Eigenvalues			Extraction Sums of Sqd. Loadings			Rotation Sums of Sqd. Loadings	
Comp.	Total	% Variance	Cumul.%	Total	% Variance	Cumul.%	Total	% Variance
1	8.742	79.5	79.5	8.742	79.5	79.5	8.093	73.6
2	1.534	13.9	93.4	1.534	13.9	93.4	2.183	19.8
3	0.334	3.0	96.5					
4	0.145	1.3	97.8					
5	0.116	1.0	98.8					
6	0.076	0.7	99.5					
7	0.029	0.3	99.8					
8	0.012	0.1	99.9					
9	0.005	0.001	99.9					
10	0.003	0.03	99.97					
11	0.003	0.03	100					
Extraction method: Principal component analysis.								

Table 5: Table of total variance explained (Sadovski, A. et al. 2010).

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